

Name: _____ ()

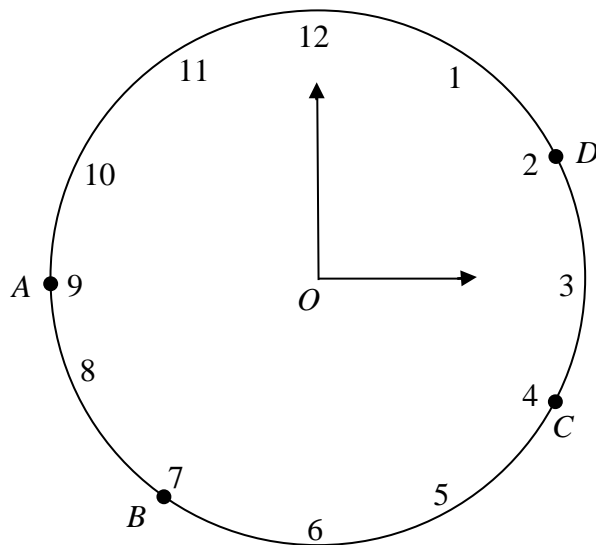
Date: _____

Class: _____

Properties of circles

Question 1

The diagram shows the face of a clock with centre O .

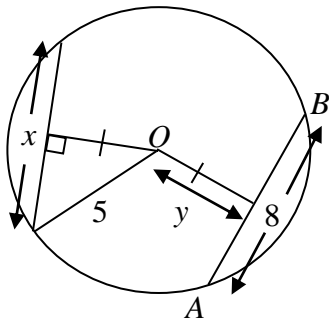


Calculate

- (a) $\angle BOC$,
- (b) $\angle BAC$,
- (c) $\angle ABC$,
- (d) $\angle ADC$.

Question 2

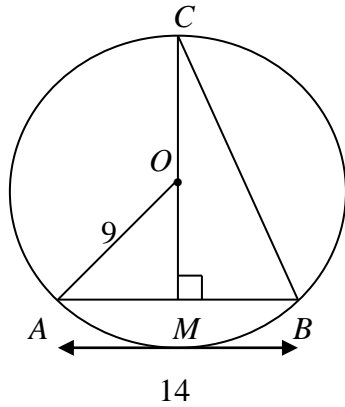
In the diagram, O is the centre of the circle with a radius of 5 cm and chord $AB = 8$ cm.



Find

- (a) x ,
- (b) y .

Question 3

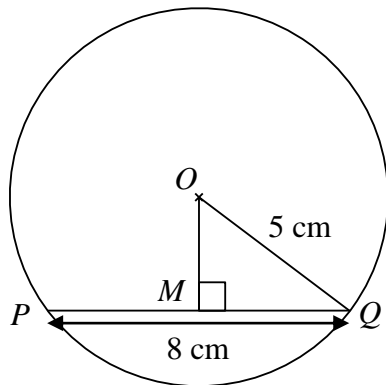


In the diagram, O is the centre of the circle of radius 9 cm and $AB = 14$ cm. Find

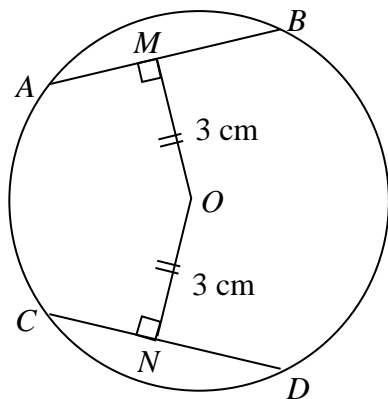
- (a) the length of OM ,
- (b) the length of CB ,
- (c) $\angle MCB$.

Question 4

The diagram shows a circle of radius 5 cm. It has a chord PQ of length 8 cm. Calculate the length of OM .



Question 5

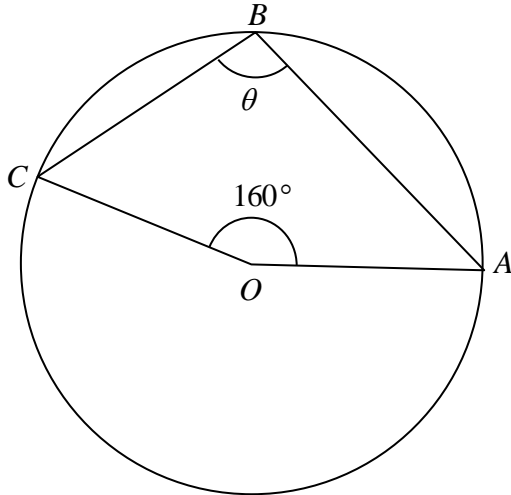


The figure shows a circle with centre O . $AB = 8$ cm and $OM = ON = 3$ cm.

- (a) Find the length of CN .
- (b) Find the radius of the circle.
- (c) Calculate $\angle OCN$.

Question 6

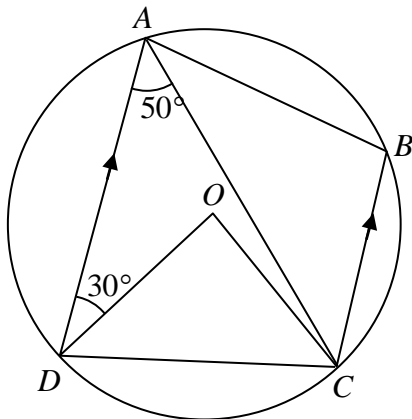
In the diagram, O is the centre of the circle. Find θ .



Question 7

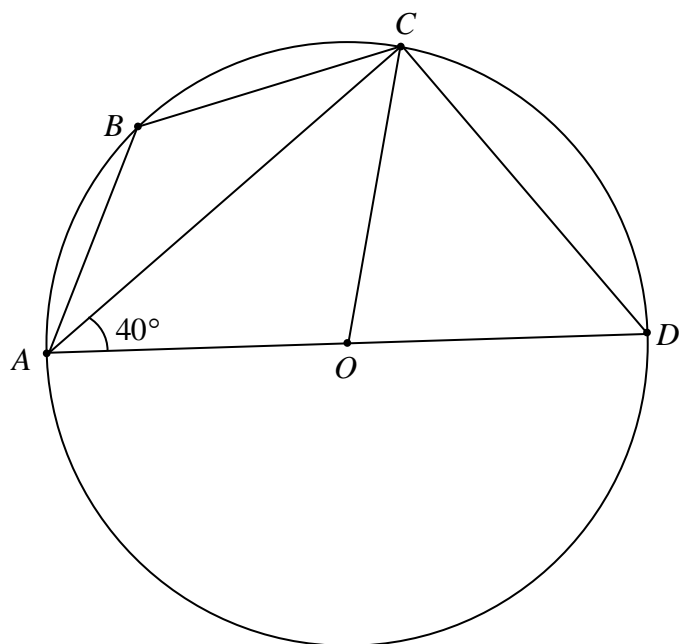
In the diagram, O is the centre of the circle $ABCD$. AD is parallel to BC . Given that $\angle CAD = 50^\circ$ and $\angle ADO = 30^\circ$, find

- (a) $\angle OCD$.
- (b) $\angle OCA$.
- (c) $\angle ABC$.
- (d) $\angle BAC$.



Question 8

In the diagram below, O is the centre of the circle and $\angle CAO = 40^\circ$.

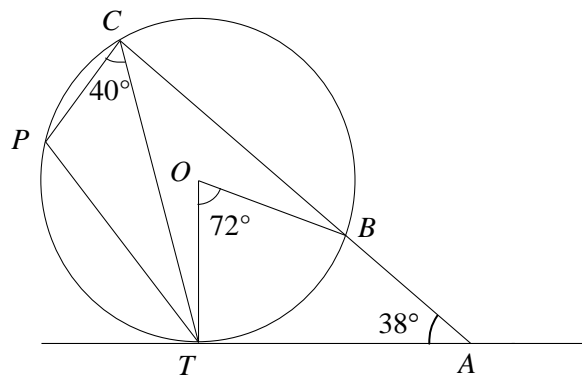


Find

- (a) $\angle COD$,
- (b) $\angle CDO$,
- (c) $\angle ABC$.

Question 9

In the figure, TA is a tangent to the circle whose centre is O and ABC is a straight line. If $\angle TOB = 72^\circ$, $\angle TAB = 38^\circ$ and $\angle PCT = 40^\circ$.



Calculate

- (a) $\angle OBA$.
- (b) $\angle OTC$.
- (c) $\angle CTP$.

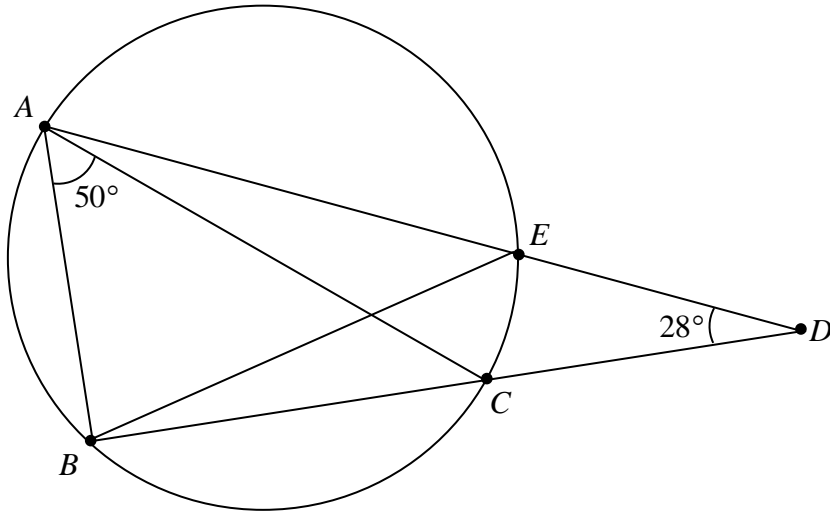
Question 10

In the diagram, AC is a diameter of the circle. The chords AE and BC meet at D when produced. Given that $\angle CDE = 28^\circ$ and $\angle BAC = 50^\circ$.

Find

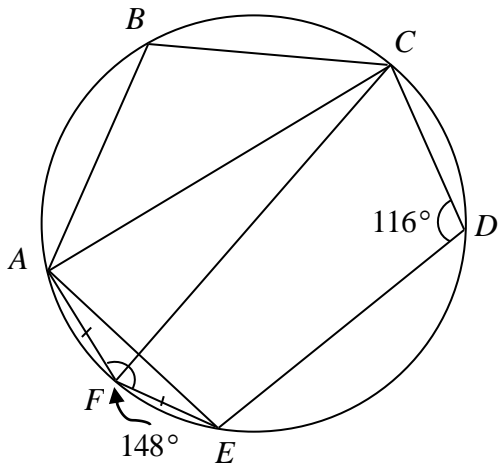
- (a) $\angle ACB$,

- (b) $\angle AEB$,
- (c) $\angle CBE$.



Question 11

$ABCDEF$ is a hexagon inscribed in a circle.

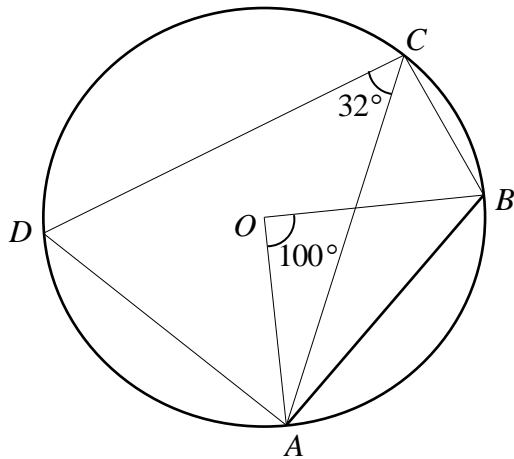


Given that $\angle AFE = 148^\circ$, $\angle CDE = 116^\circ$ and $FA = FE$, find

- (a) $\angle ACF$,
- (b) $\angle ABC$.

Question 12

In the diagram, O is the centre of the circle, $\angle AOB = 100^\circ$ and $\angle ACD = 32^\circ$.



Find, stating your reasons clearly,

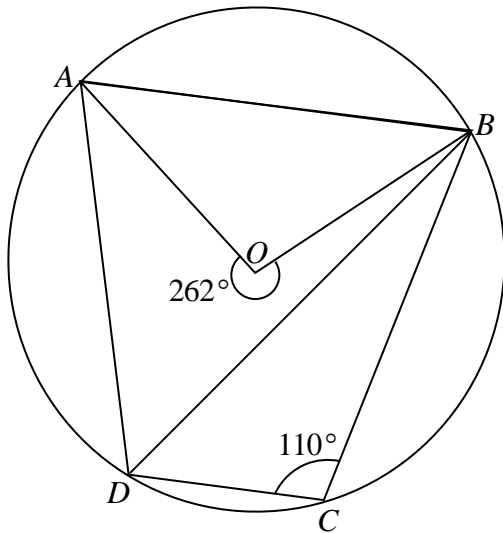
- (a) $\angle ACB$,
- (b) $\angle DAB$,
- (c) $\angle DAO$.

Question 13

In the diagram, A, B, C and D are points on a circle, centre O . Reflex $\angle AOB = 262^\circ$ and

$\angle DCB = 110^\circ$. Stating your reasons clearly, find

- (a) $\angle ADB$.
- (b) $\angle DAO$.

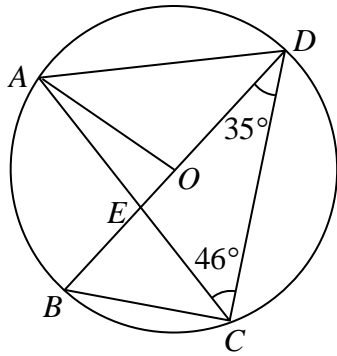


Question 14

Given that O is the centre of circle, $\angle ACD = 46^\circ, \angle BDC = 35^\circ$, and BOD is the diameter

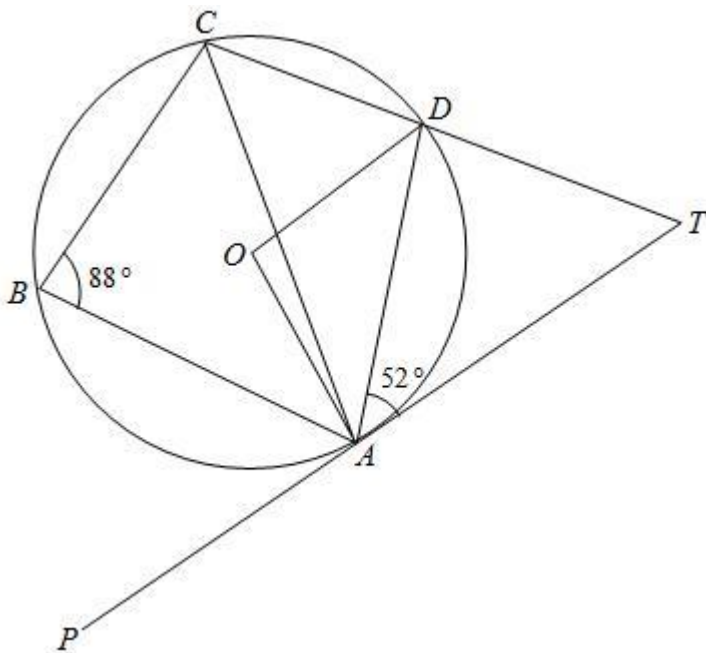
of the circle. Find

- (a) $\angle AOD$,
- (b) $\angle BCE$,
- (c) $\angle CAD$.



Question 15

In the figure, O is the centre of the circle, $\angle ABC = 88^\circ$ and $\angle DAT = 52^\circ$. The tangent PAT meets the line CD produced at T .

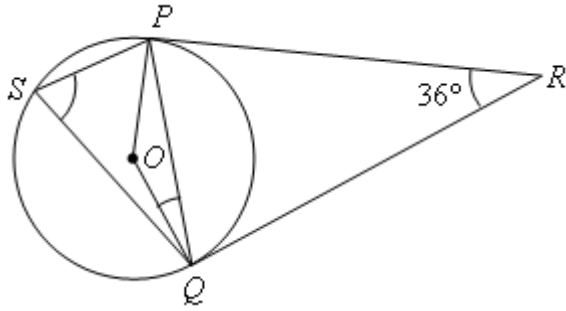


Calculate

- (a) $\angle OAD$,
- (b) $\angle AOD$,
- (c) $\angle ACD$,
- (d) $\angle ODC$.

Question 16

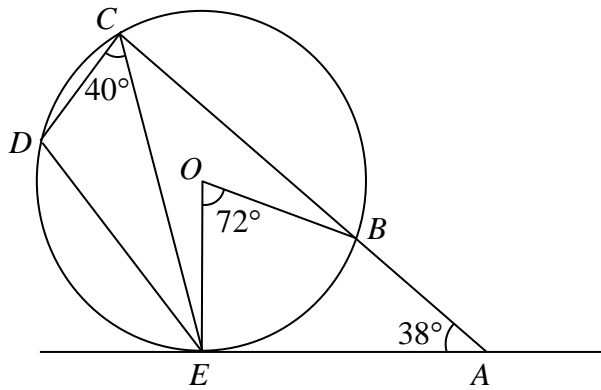
O is the centre of the circle through P , Q and S . PR and QR are tangents to the circle.



- Find $\angle POQ$.
- Find $\angle PSQ$.
- Give a reason for your answer.

Question 17

In the figure, AE is a tangent to the circle whose centre is O and ABC is a straight line. $\angle BAE = 38^\circ$, $\angle BOE = 72^\circ$ and $\angle DCE = 40^\circ$.

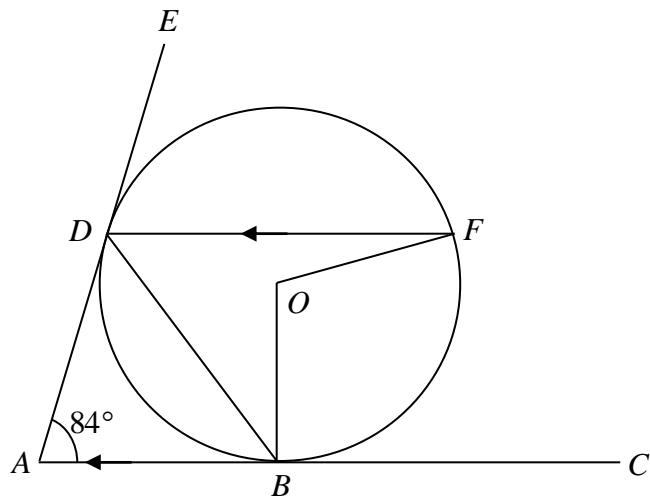


Find, stating your reasons clearly

- $\angle ECB$,
- $\angle OEC$,
- $\angle CED$,
- $\angle OBA$.

Question 18

In the figure below, ABC and ADE are tangents to the circle with centre O .



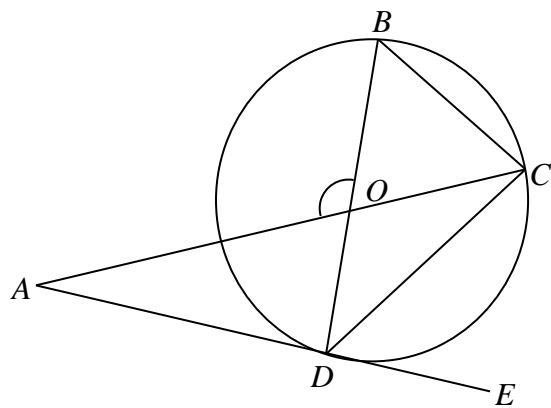
Given the DF is parallel to ABC and $\angle DAB = 84^\circ$, stating your reasons clearly, find

- (a) $\angle ADB$.
- (b) $\angle BOF$.
- (c) $\angle FBC$.
- (d) $\angle DFO$.

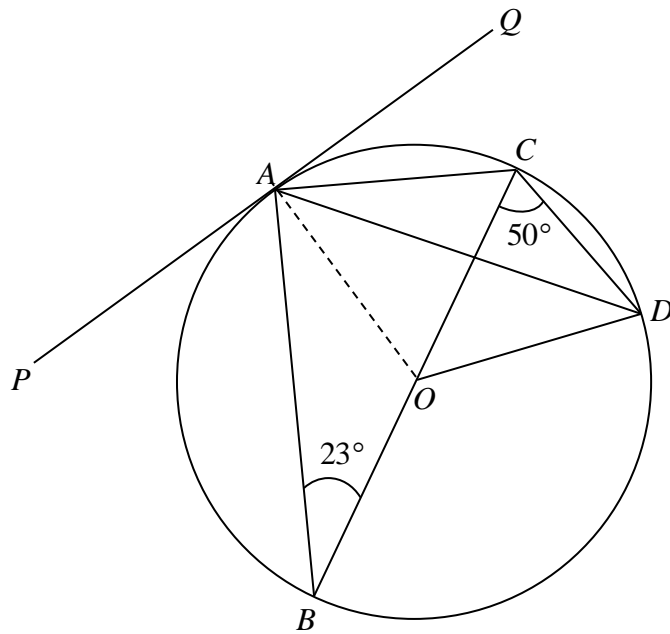
Question 19

In the diagram, O is the centre of the circle. The tangent of the circle at D meets CO produced at A . Given that $\angle AOB = 132^\circ$, calculate

- (a) $\angle ACD$.
- (b) $\angle DBC$.
- (c) $\angle CAD$.
- (d) $\angle CDE$.



Question 20

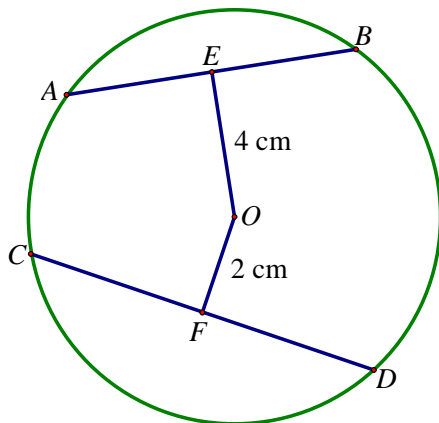


In the diagram, BC is the diameter of a circle of centre O . PQ is tangent to the circle at A .

Given that $\angle ABC = 23^\circ$ and $\angle OCD = 50^\circ$, find, stating your reasons clearly,

- (a) $\angle ADC$,
- (b) $\angle ADO$,
- (c) $\angle QAD$.

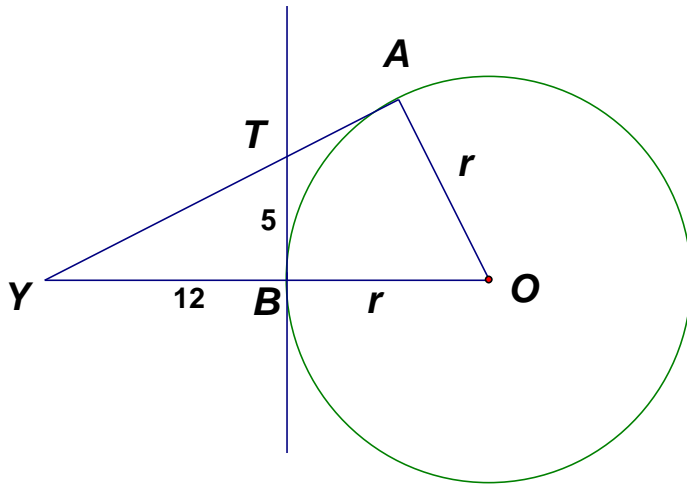
Question 21



The diagram shows four points A , B , C and D on the circumference of a circle centre O . The points E and F are mid-points of chords AB and CD respectively. It is given that $AB = 8$ cm, $OE = 4$ cm and $OF = 2$ cm, calculate the length of the chord CD .

Question 22

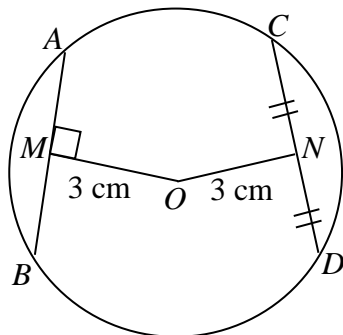
TA and TB are tangents to a circle centre O and AT produced meets OB at Y . $YB = 12$ cm, $TB = 5$ cm and $OA = r$ cm.



- Write down the length of TA .
- Calculate the length of TY .
- Without solving (d), find the numerical value of $\frac{OA}{AY}$.
- Calculate the value of r .

Question 23

The figure below shows a circle with centre O and chords AB and CD . $AB = 5$ cm and $OM = ON = 3$ cm.

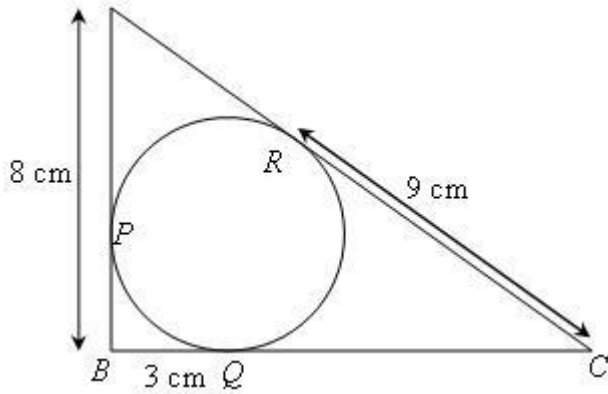


Calculate

- the length of CN .
- the radius of the circle.

Question 24

In the given diagram, AB , BC and AC are tangents to the circle of radius 3cm at P , Q and R respectively. AB is 8 cm, RC is 9 cm and BQ is 3 cm. AB is perpendicular to BC .



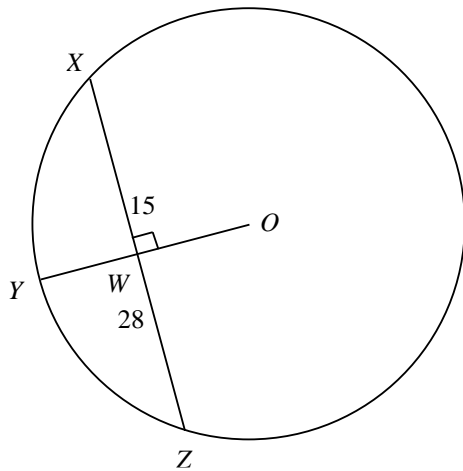
Calculate the value of

- (a) AR ,
- (b) CQ ,
- (c) the perimeter of $\triangle ABC$.

Question 25

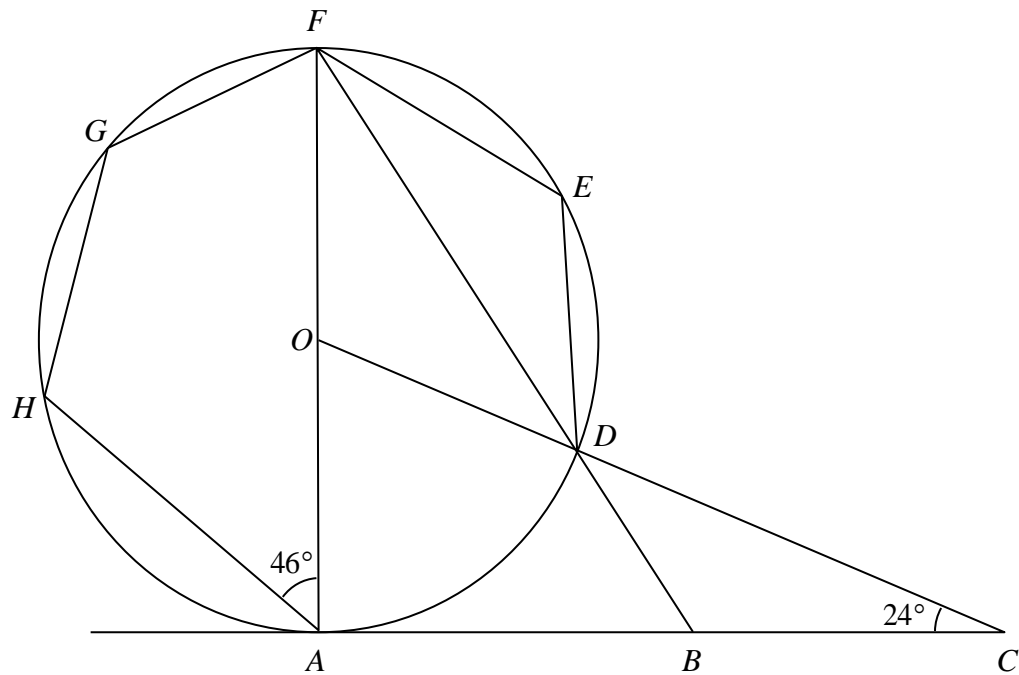
In the diagram, a circle with centre O has radius $OY = 15$ cm. A chord XZ of length 28 cm cuts OY perpendicularly at W . Find

- (a) the length of WY ,
- (b) the length of YZ ,
- (c) the area of $\triangle OXW$.



Question 26

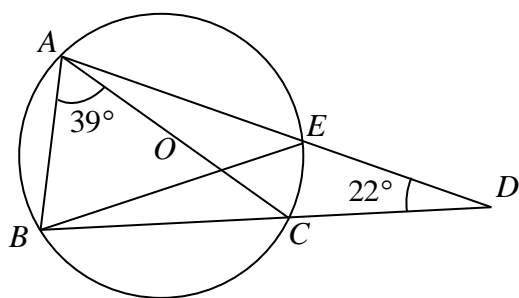
The diagram shows a circle with centre O . AC is a tangent to the circle. AOF , FDB and ODC are straight lines. Given that $\angle ACO = 24^\circ$ and $\angle HAF = 46^\circ$.



- (a) Write down why $\angle FAB = 90^\circ$.
- (b) Calculate
- (i) $\angle HEF$.
 - (ii) $\angle AFB$.
 - (iii) $\angle HGF$.
 - (iv) $\angle FED$.

Question 27

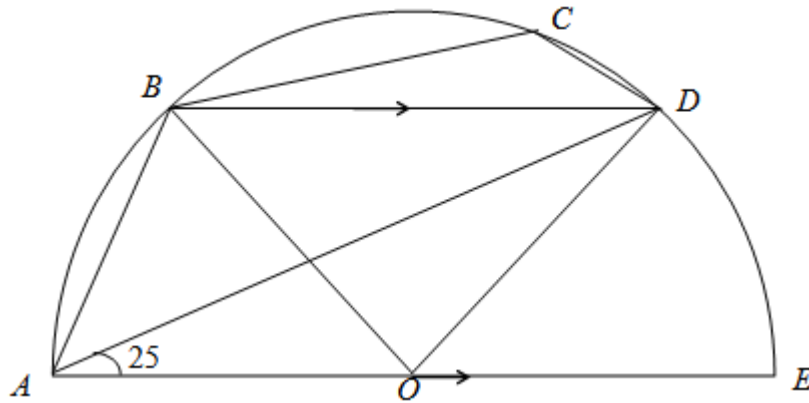
In the diagram, AC is a diameter of the circle with center O . The chords AE and BC meet at D when produced. Given that $\angle CDE = 22^\circ$ and $\angle BAC = 39^\circ$.



Find

- (a) $\angle ACB$,
- (b) $\angle AEB$,
- (c) $\angle CBE$.

Question 28

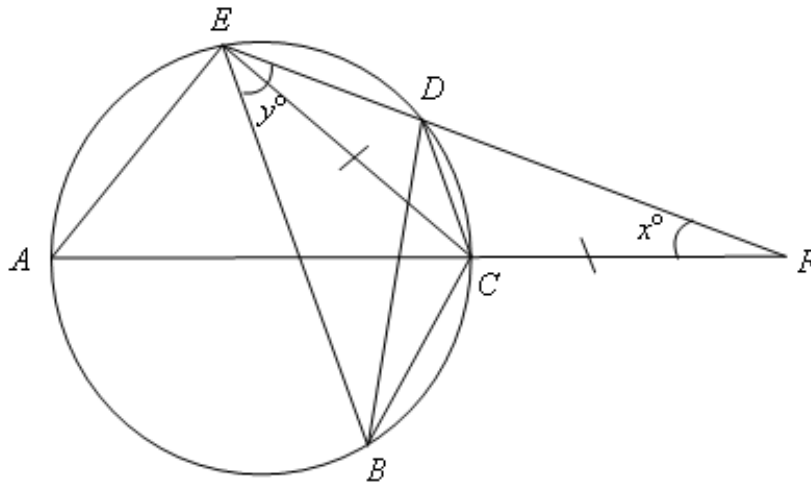


In the figure above, AE is the diameter of the circle, centre O , $BD \parallel AE$ and angle $DAO = 25^\circ$. Find, giving reason(s) for each answer,

- (a) angle DOE ,
- (b) angle BOD ,
- (c) angle BAD ,
- (d) angle ABD .
- (e) angle BCD .

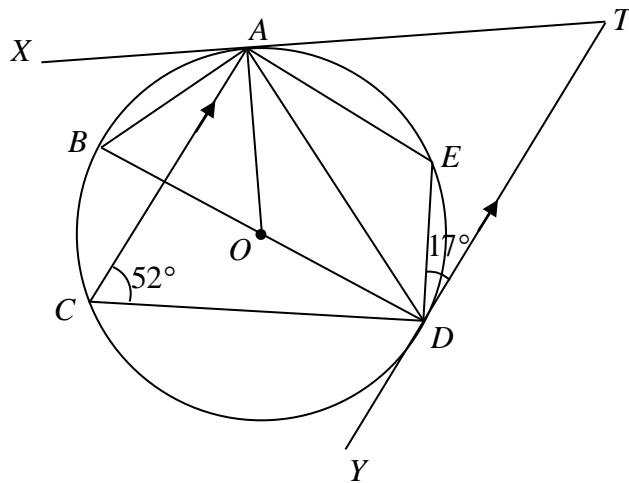
Question 29

In the diagram, AC is a diameter of the circle $ABCDE$. AC and ED are extended to meet at F . Given that $CE = CF$, $\angle AFD = x^\circ$ and $\angle BED = y^\circ$.



- (a) Explain with reasons why $\angle DBC = x^\circ$.
- (b) Find, in terms of x and/or y ,
 - (i) $\angle ACD$.
 - (ii) $\angle ACE$.
 - (iii) $\angle BCD$.

Question 30



XT and YT are tangents to the circle at A and D respectively. O is the centre of the circle,

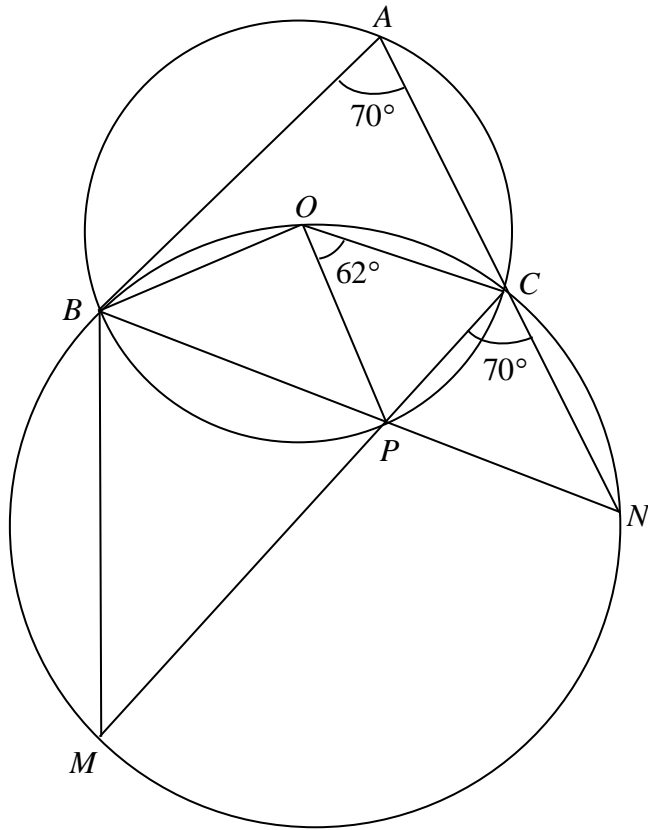
CA is parallel to DT , $\angle ACD = 52^\circ$ and $\angle EDT = 17^\circ$.

Find

- (a) $\angle ABD$.
- (b) $\angle AOD$.
- (c) $\angle ADO$.
- (d) $\angle ODC$.
- (e) $\angle CAE$.

Question 31

In the diagram, O is the centre of circle $ABPC$. $\angle BAC = \angle MCN = 70^\circ$ and $\angle POC = 62^\circ$. The larger circle $BMNC$ passes through O . ACN , BPN and CPM are straight lines.

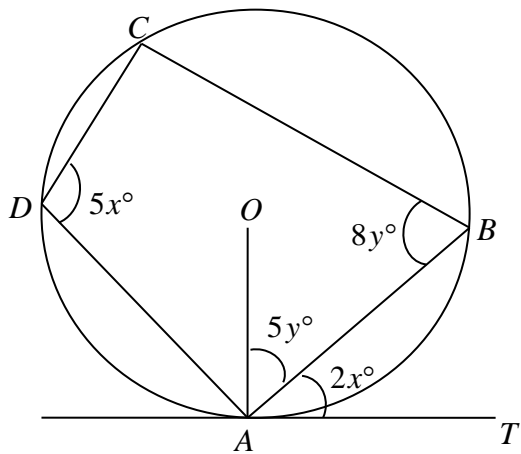


Give reasons and find the value of

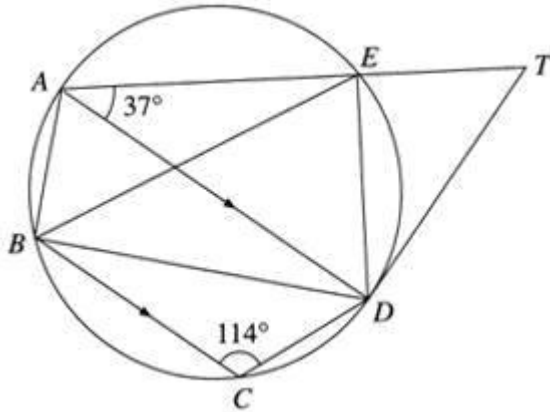
- (a) reflex $\angle BOC$.
- (b) $\angle ABP$.
- (c) $\angle MBN$.
- (d) $\angle BMP$.
- (e) $\angle ACO$.

Question 32

A, B, C, D are four points on the circle with centre O . AT is a tangent at A . Form two simultaneous equations in x and y and hence find the value of x and y .



[Question 33](#)



The points A, B, C, D and E lie on the circle. AD is the diameter of the circle. DT is a tangent to the circle. AET is a straight line and AD is parallel to BC . $\angle BCD = 114^\circ$ and $\angle DAE = 37^\circ$. Find

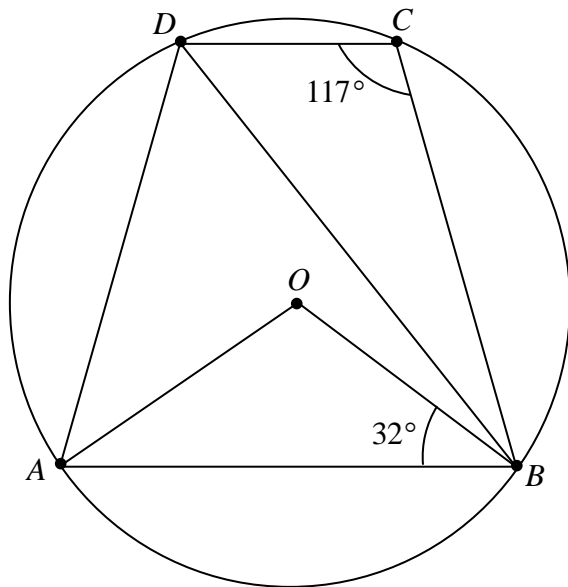
- (a) $\angle ADE$,
- (b) $\angle ATD$,
- (c) $\angle DBE$,
- (d) $\angle BAD$,
- (e) $\angle BDC$.

[Question 34](#)

In the diagram, O is the centre of the circle. A, B, C and D are points on the circumference of the circle. Given that $\angle ABO = 32^\circ$ and $\angle BCD = 117^\circ$.

Find

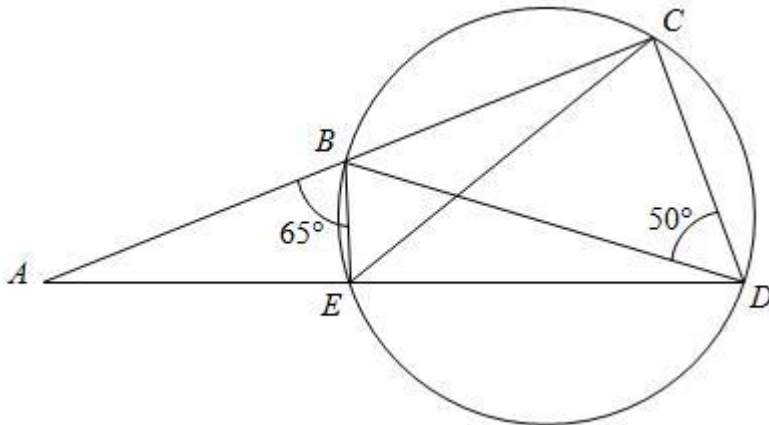
- (a) $\angle BAD$,
- (b) $\angle ADB$,
- (c) $\angle OBD$.



[Question 35](#)

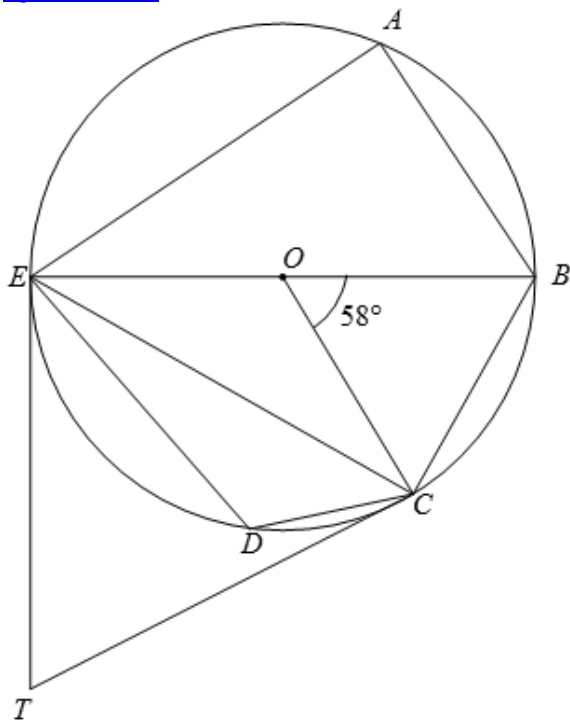
In the diagram, ABC and AED are straight lines. B, C, D and E lie on a circle.

$\angle ABE = 65^\circ$ and $\angle BDC = 50^\circ$.



- Find $\angle CDE$
- Find $\angle BCE$
- Name the triangle which is similar to triangle ABD , explaining why the two triangles are similar.

[Question 36](#)



The diagram shows a circle $ABCDE$, centre O . CT and ET are tangents to the circle. BE is a diameter and angle $BOC = 58^\circ$.

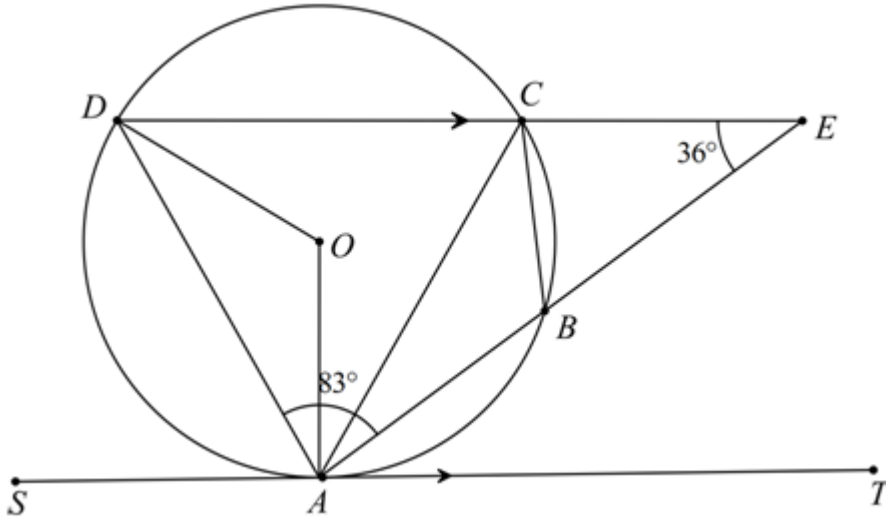
- Explain why angle EAB is a right angle.
- State two other angles which are right angle.
- Find, giving reasons for each answer,
 - angle EBC ,

- (ii) angle BEC ,
- (iii) angle EDC ,
- (iv) angle CET ,

(d) Given that OE is 3 cm, find the area of quadrilateral $OETC$.

Question 37

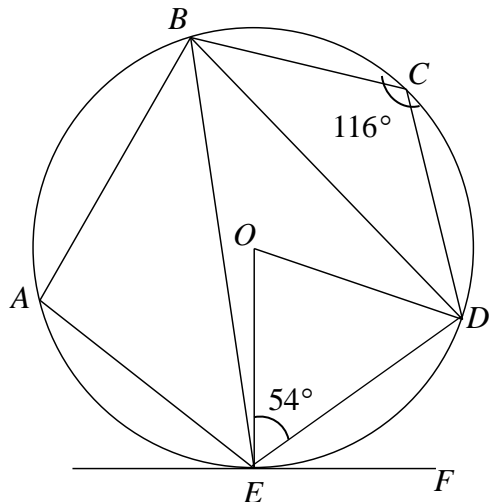
In the diagram, A, B, C and D are points on the circumference of the circle centre O . ABE and DCE are straight lines and ST is a tangent to the circle at A . ST is parallel to DE , $\angle CEB = 36^\circ$ and $\angle DAB = 83^\circ$.



- (a) Calculate
 - (i) angle OAB ,
 - (ii) angle ABC ,
 - (iii) angle AOD .
- (b) Show that ACD is an isosceles triangle.

Question 38

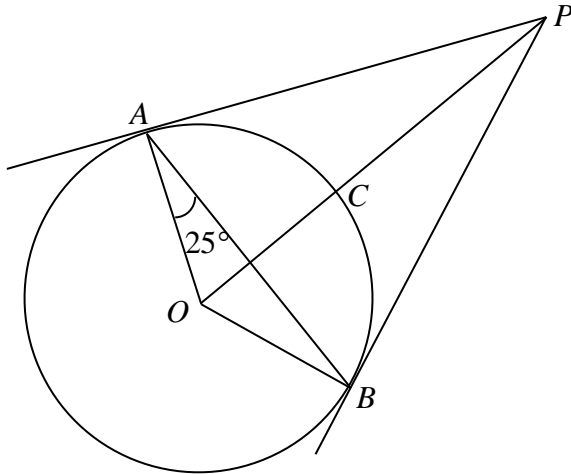
In the diagram A, B, C, D and E lie on a circle centre O . Angle $BCD = 116^\circ$ and angle $OED = 54^\circ$. EF is tangent to the circle at E .



Find

- (a) angle DOE ,
- (b) angle DBE ,
- (c) angle DEF ,
- (d) angle OEB ,
- (e) angle BAE .

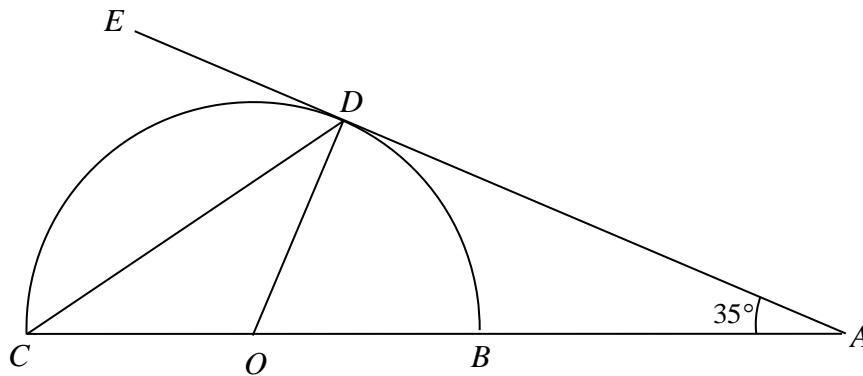
Question 39



- (a) The diagram shows a circle ABC , with centre O . AP and BP are tangents to the circle. $\angle OAB = 25^\circ$. Stating your reasons clearly, find
 - (i) $\angle AOB$.
 - (ii) $\angle PAB$.
 - (iii) $\angle APB$.
- (b) Given that the radius of the circle is 5 cm and $OP = 16$ cm, find the length of AP .

Question 40

The diagram shows a semicircle with centre O . The line AE is tangent to the circle at D and $\angle CAE = 35^\circ$.



- (a) Calculate
 - (i) angle BOD ,
 - (ii) angle OCD ,

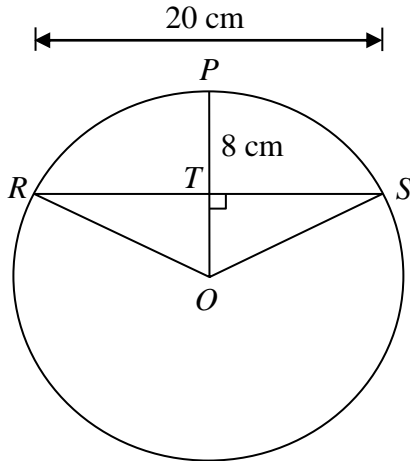
(iii) angle CDE .

(b) Explain why AO is the diameter of a circle passing through the points A , O and D .

Question 41

The perpendicular bisector of a chord, RS , cuts it at T and the circumference of the circle

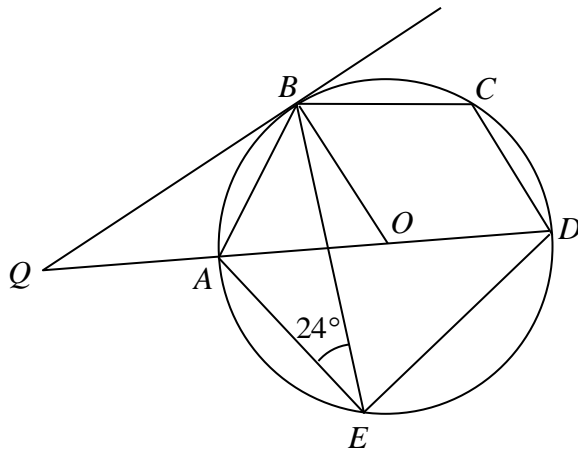
at P . If $RS = 20$ cm and $TP = 8$ cm, find the radius of the circle.



Question 42

The points A , B , C , D and E lie on a circle, centre O . AD is a diameter of the circle.

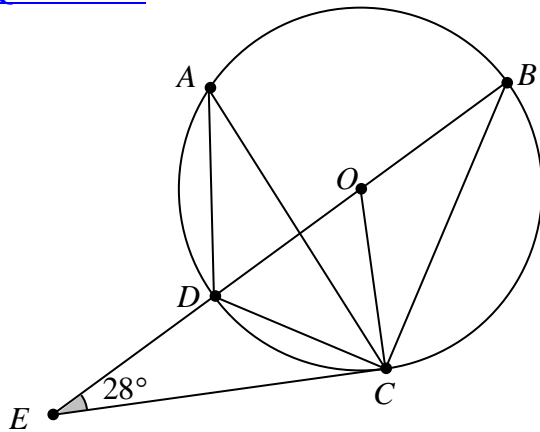
BQ is a tangent to the circle, $\angle AEB = 24^\circ$ and QAD is a straight line.



Calculate, stating your reasons clearly,

- (a)
 - (i) $\angle BAD$.
 - (ii) $\angle BED$.
 - (iii) $\angle BQA$.
 - (iv) reflex $\angle BOD$.
 - (v) $\angle ABQ$.
- (b) If $\angle DAE = x^\circ$, show that $\angle QBE = (114 - x)^\circ$.
- (c) Given that $\angle ABE = 43^\circ$, determine whether the lines BO and AE are parallel. Give a reason for your answer.

Question 43



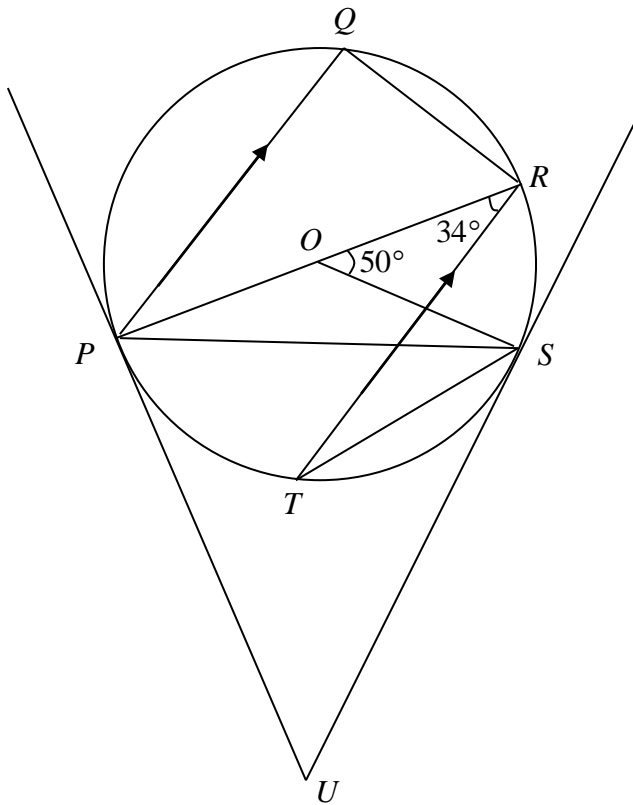
In the diagram, O is the centre of the circle. A , B , C and D are points on the circle where

BOD is a straight line. The line CE is a tangent to the circle at point C . $\angle OEC = 28^\circ$. Find the value of

- (a) $\angle OCE$,
- (b) $\angle ODC$,
- (c) $\angle DAC$,
- (d) $\angle DBC$,
- (e) $\angle ACB$ if $\angle ABD = 35^\circ$.

Question 44

In the diagram below, P , Q , R , S and T are points on a circle with centre at O . PR is a diameter of the circle, and PQ is parallel to TR . The tangent to the circle at P and the tangent to the circle at S meet at U .

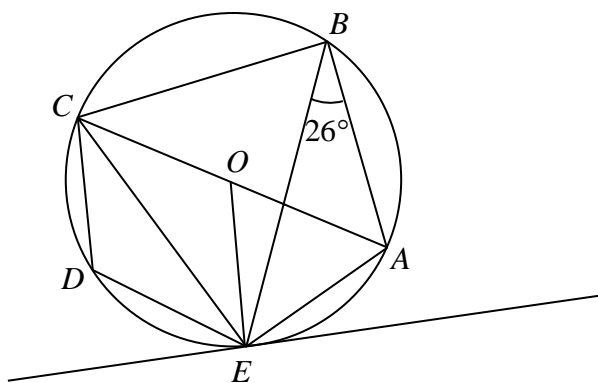


Given that $\angle PRT = 34^\circ$, $\angle ROS = 50^\circ$ and $OP = 5$ cm.

Find

- $\angle RTS$,
- $\angle OST$,
- $\angle QRP$,
- the length of PU .

[Question 45](#)



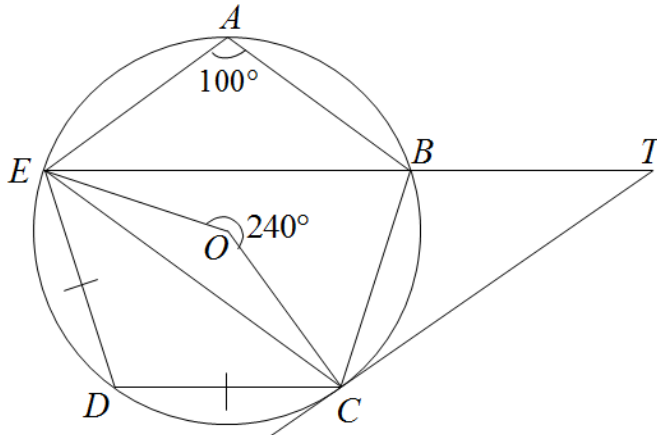
The points A, B, C, D and E lie on a circle, centre O . AC is a diameter of the circle. EQ is a tangent to the circle at E and $\angle ABE = 26^\circ$.

- Stating your reasons clearly, find
 - $\angle ACE$.
 - $\angle CBE$.
 - $\angle CDE$.

- (iv) $\angle AEQ$.
- (b) Given that $\angle AEB = 38^\circ$, determine whether the lines EO and AB are parallel. Give a reason for your answer.

Question 46

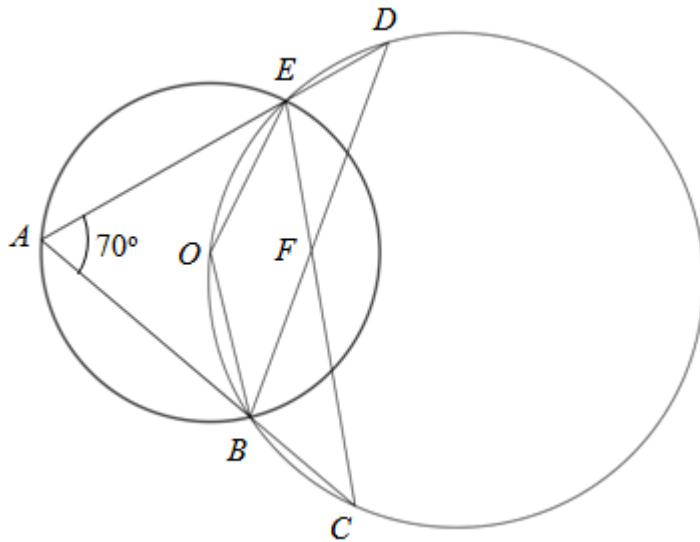
In the diagram, O is the centre of circle $ABCDE$ where $DE = DC$. Reflex angle $EOC = 240^\circ$ and angle $EAB = 100^\circ$.
(You must not assume CT is a tangent to the circle at C .)



- (a) Find, giving reasons for each answer,
- (i) angle DCE ,
 - (ii) angle CBE ,
 - (iii) angle CEB .
- (b) Given that $\angle CTE = 20^\circ$ and EBT is a straight line, show that CT is a tangent to the circle at C .

Question 47

The diagram shows two circles with the bigger circle passing through the centre, O , of the smaller circle. The circles intersect at B and E . A lies on the smaller circle while C and D lie on the bigger circle. AED and ABC are straight lines. CE and BD intersect at F and $\angle BAE = 70^\circ$.

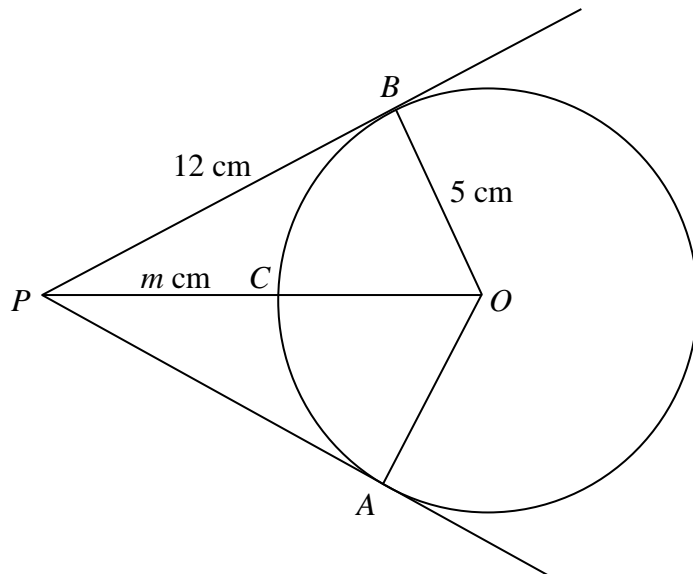


- (a) Find, stating the reasons clearly,
- (i) $\angle BOE$,
 - (ii) $\angle BDE$,
 - (iii) $\angle ABD$,
 - (iv) $\angle CED$,
 - (v) $\angle DFC$.
- (b) A point G lies on the same side of BE as A . Given that $\angle BGE = 75^\circ$, explain whether point G lies inside, outside or on the circumference of the smaller circle.

Question 48

In the diagram, PA and PB are tangents to the circle with centre, O . PCO is a straight line where C is a point on the circle. Find the

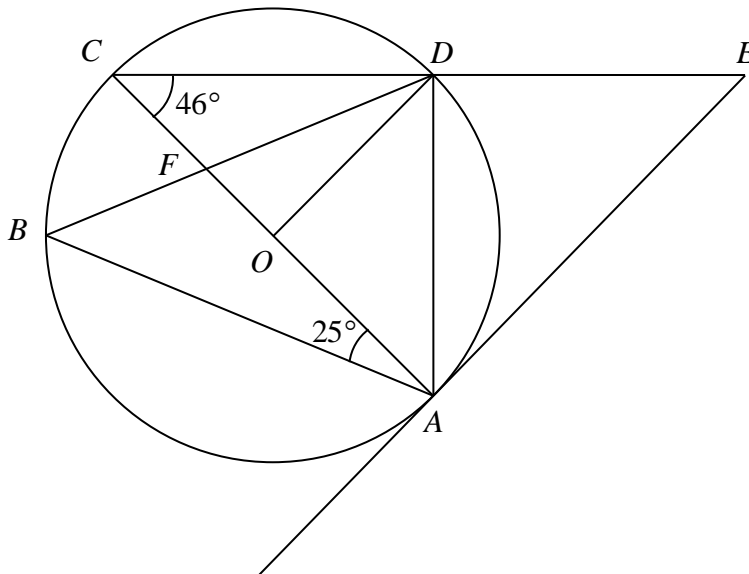
- (a) value of m , where $PC = m$ cm.
- (b) angle BPA .



Question 49

In the diagram below, O is the centre of the circle and points A, B, C and D lie on the

circumference of the circle. The tangent at A meets CD produced at E . The line AC passes through O and lines AC intersects BD at point F . It is also given that $\angle ACD = 46^\circ$ and $\angle CAB = 25^\circ$.

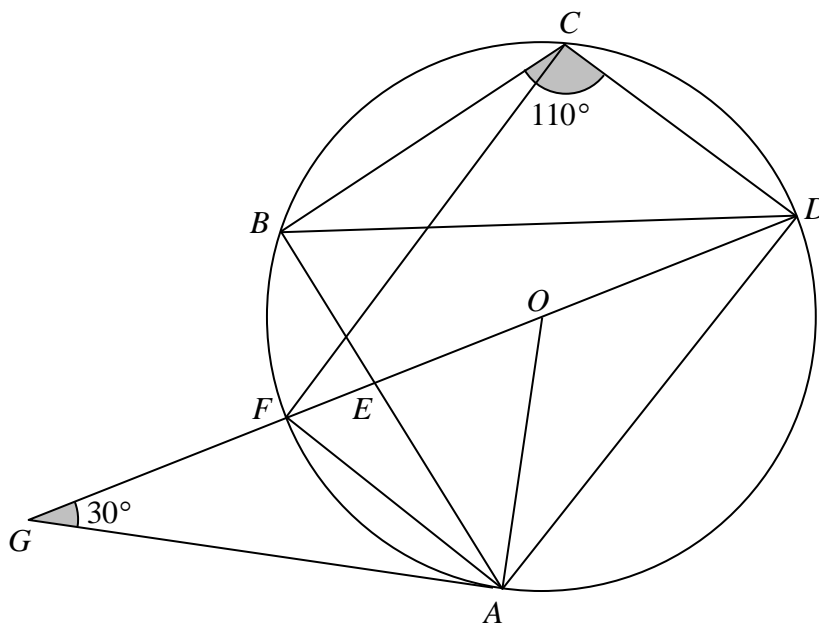


Stating your reasons clearly, find

- (a) $\angle ADO$.
- (b) $\angle OFD$.
- (c) $\angle AEC$.

Question 50

In the diagram, O is the centre of the circle, $DEFG$ and AEB are straight lines and GA is a tangent to the circle. $\angle AGD = 30^\circ$ and $\angle BCD = 110^\circ$.



- (a) Find, stating your reasons clearly,

- (i) $\angle GOA$,
- (ii) $\angle FDA$,
- (iii) $\angle BAO$,
- (iv) $\angle BDF$.

(b) Show that $\triangle GAO$ is congruent to $\triangle DAF$.

(c) Given that a point X lies on the circumference, on the same side of semicircle as point B , such that $GX = GA$. Describe the possible location of X stating the reasons clearly.

[Answer Key]

Question 1

- (a) 90° (b) 45° (c) 105° (d) 75°

Question 2

- (a) 8 (b) 3

Question 3

Question 4

Question 5

Question 6

- 100°

Question 7

- (a) 40° (b) 20° (c) 110° (d) 20°

Question 8

- (a) 80° (b) 50° (c) 130°

Question 9

- (a) 160° (b) 16° (c) 34°

Question 10

- (a) 40° (b) 40° (c) 12°

Question 11

- (a) 16° (b) 96°

Question 12

- (a) 50° (b) 98° (c) 58°

Question 13

- (a) 49° (b) 29°

Question 14

- (a) 92° (b) 44° (c) 55°

Question 15

- (a) 38° (b) 104° (c) 52° (d) 54°

Question 16

- (a) 144° (b) 72°
(c) $\angle PSQ = 72^\circ$ because angle at centre equals twice angle at circumference.

Question 17

- (a) 36° (b) 16° (c) 34° (d) 160°

[Question 18](#)

- (a) 48° (b) 96° (c) 48° (d) 6°

[Question 19](#)

- (a) 24° (b) 66° (c) 42° (d) 66°

[Question 20](#)

- (a) 23° (b) 27° (c) 63°

[Question 21](#)

[Question 22](#)

[Question 23](#)

- (a) 2.5 cm (b) 3.91 cm

[Question 24](#)

- (a) 5 cm (b) 9 cm (c) 34 cm

[Question 25](#)

[Question 26](#)

- (a) AC is a tangent to circle therefore $\angle FAB = 90^\circ$
(b) (i) 46° (ii) 33° (iii) 134° (iv) 123°

[Question 27](#)

- (a) 51° (b) 51° (c) 29°

[Question 28](#)

- (a) 50° (b) 80° (c) 40° (d) 115° (e) 140°

[Question 29](#)

- (b) (i) $90 - x$ (ii) $2x$ (iii) $180 - y$

[Question 30](#)

- (a) 52° (b) 104° (c) 38° (d) 38° (e) 69°

[Question 31](#)

- (a) 220° (b) 70° (c) 70° (d) 40° (e) 51°

[Question 32](#)

$x = 20, y = 10$

[Question 33](#)

- (a) 53° (b) 53° (c) 37° (d) 66° (e) 42°

[Question 34](#)

- (a) 63° (b) 58° (c) 27°

Question 35

- (a) 65° (b) 15° (c) Triangle AEC is similar to triangle ABD by AAA.

Question 36

- (a) Angle in semicircle subtended by the diameter
(b) Angle BET , angle OCT and Angle BCE
(c) (i) 61° (ii) 29° (iii) 119° (iv) 61° (d) 16.2 cm^2

Question 37

- (a) (i) 54° (ii) 119° (iii) 122°

Question 38

- (a) 72° (b) 36° (c) 36° (d) 10° (e) 100°

Question 39

- (a) (i) 130° (ii) 65° (iii) 50° (b) 15.2 cm

Question 40

- (a) (i) 55° (ii) 27.5° (iii) 62.5° (b) 90° (right \angle in semi - circle)

Question 41

10.25 cm or $10\frac{1}{4} \text{ cm}$

Question 42

- (a) (i) 66° (ii) 66° (iii) 42° (iv) 228° (v) 24°
(c) Not parallel as $\angle EBQ \neq \angle AEB$

Question 43

- (a) 90° (b) 59° (c) 31° (d) 31° (e) 55°

Question 44

- (a) 25° (b) 59° (c) 56° (d) 10.7 cm

Question 45

- (a) (i) 26° (ii) 64° (iii) 116° (iv) 26°
(b) EO and AB are parallel

Question 46

- (a) (i) 30° (ii) 60° (iii) 40°

Question 47

- (a) (i) 140° (ii) 40° (iii) 70° (iv) 110° (v) 150°
(b) Any point X on the same side of BE as A , lying on the circumference of the circle will form an angle $\angle BXE = 70^\circ$ (angles in same segment). Since $\angle BGE = 75^\circ$ which is larger than 70° , G lies in the circle.

Question 48

- (a) 8 (b) 45.2°

Question 49

- (a) 44° (b) 71° (c) 44°

Question 50

- (a) (i) 60° (ii) 30° (iii) 40° (iv) 20°
(c) For $GX = GA$, GX must be a tangent to the circle (tangents from ext. point) and X lies on the circumference where $\angle GOX = 60^\circ$ or $\angle OGX = 30^\circ$.

[Solution]

Question 1

(a)

$$5 \text{ minutes} : 30^\circ$$

$$15 \text{ minutes} : 90^\circ$$

Question 6

$$\begin{aligned} \text{Reflex } \angle COA &= 360^\circ - 160^\circ \\ &= 200^\circ \quad (\angle \text{s at a point}) \end{aligned}$$

$$\begin{aligned} \angle \theta &= 200 \div 2 \\ &= 100^\circ \quad (\angle \text{ at centre} = 2 \angle \text{ at circumference}) \end{aligned}$$

Question 7

(a) $\angle DOC = 100^\circ$ (\angle at centre = $2 \times \angle$ at circumference)

$$\begin{aligned} \angle OCD &= \frac{180^\circ - 100^\circ}{2} \\ &= 40^\circ \end{aligned}$$

(b)

$$\begin{aligned} \angle OCA &= 180^\circ - 50^\circ - 30^\circ - 40^\circ - 40^\circ \\ &= 20^\circ \end{aligned}$$

(c)

$$\begin{aligned} \angle ABC &= 180^\circ - 30^\circ - 40^\circ \\ &= 110^\circ \end{aligned}$$

(d)

$$\begin{aligned} \angle BAC &= 180^\circ - 110^\circ - 50^\circ \\ &= 20^\circ \end{aligned}$$

Question 8

(a) $\angle COD = 80^\circ$ (\angle at centre = twice \angle at circumference)

(b)

$$\begin{aligned} \angle CDO &= 90^\circ - 40^\circ \\ &= 50^\circ \quad (\angle \text{ in a semicircle}) \end{aligned}$$

(c)

$$\begin{aligned} \angle COD &= 180^\circ - 50^\circ \\ &= 130^\circ \quad (\text{opp. } \angle \text{ of cyclic quad.}) \end{aligned}$$

Question 9

(b)

$$\begin{aligned}\angle TCB &= \frac{72^\circ}{2} \\ &= 36^\circ \quad (\angle \text{ at centre} = 2 \angle \text{ at circum.})\end{aligned}$$

$$\angle CTA = 106^\circ \quad (\angle \text{ sum of } \Delta)$$

$$\angle OTC = 16^\circ$$

(c) $\angle PTB = 104^\circ$ (opp. \angle s, cyclic. *quadrilateral*)

$$\angle OTB = 54^\circ \quad (\text{isos. } \Delta)$$

$$\angle CTP = 34^\circ$$

Question 11

(a)

$$\begin{aligned}\angle FEA &= (180^\circ - 148^\circ) \div 2 \quad (\angle \text{s of isos. } \Delta) \\ &= 16^\circ\end{aligned}$$

$$\angle ACF = 16^\circ \quad (\angle \text{s in same segment})$$

(b)

$$\begin{aligned}\angle CFE &= 180^\circ - 116^\circ \quad (\text{opp. } \angle \text{s, cyclic quad.}) \\ &= 64^\circ\end{aligned}$$

$$\begin{aligned}\angle AFC &= 148^\circ - 64^\circ \\ &= 84^\circ\end{aligned}$$

$$\begin{aligned}\angle ABC &= 180^\circ - 84^\circ \quad (\text{opp. } \angle \text{s, cyclic quad.}) \\ &= 96^\circ\end{aligned}$$

Question 12

(a)

$$\begin{aligned}\angle ACB &= \frac{100^\circ}{2} \quad (\angle \text{ at centre} = 2 \angle \text{ at circum.}) \\ &= 50^\circ\end{aligned}$$

(b)

$$\begin{aligned}\angle DAB &= 180^\circ - 32^\circ - 50^\circ \quad (\angle \text{s in the opp. segment}) \\ &= 98^\circ\end{aligned}$$

(c)

$$\begin{aligned}\angle OAB &= \frac{180^\circ - 100^\circ}{2} \quad (\text{base } \angle \text{ of isos. } \Delta) \\ &= 40^\circ\end{aligned}$$

$$\begin{aligned}\angle DAO &= 98^\circ - 40^\circ \\ &= 58^\circ\end{aligned}$$

Question 18

(a) $AD = AB$ (tan. from ext. pt.)

$$\angle ADB = 48^\circ \text{ (base } \angle\text{s of isos. } \Delta\text{)}$$

(b) $\angle BDF = 48^\circ$ (alt. \angle s, $DF \parallel AB$)

$$\angle BOF = 96^\circ \text{ (}\angle\text{s at the centre} = 2 \angle\text{s at circum.)}$$

(c) $\angle OBC = 90^\circ$ (tan. \perp rad.)

$$\angle OBF = 42^\circ \text{ (base } \angle\text{s of isos } \Delta\text{)}$$

$$\begin{aligned}\angle FBC &= 90 - 42 \\ &= 48^\circ\end{aligned}$$

(d)

$$\begin{aligned}\angle DFB &= \angle FBC \\ &= 48^\circ \text{ (alt. } \angle\text{s, } DF \parallel AB\text{)}$$

$$\begin{aligned}\angle DFO &= 48 - 42 \\ &= 6^\circ\end{aligned}$$

Question 19

(a) $\angle DOC = 132^\circ$ (vert. opp. \angle s)

$$\begin{aligned}\angle ACD &= \frac{180^\circ - 132^\circ}{2} \text{ (isos. } \Delta\text{)} \\ &= 24^\circ\end{aligned}$$

(b)

$$\begin{aligned}\angle DBC &= \frac{1}{2}(132^\circ) \text{ (}\angle\text{ at the circum.)} \\ &= 66^\circ\end{aligned}$$

(c)

$$\begin{aligned}\angle CAD &= 180^\circ - (90^\circ + 24^\circ + 24^\circ) \text{ (sum of } \angle\text{s in a } \Delta\text{)} \\ &= 42^\circ\end{aligned}$$

(d)

$$\begin{aligned}\angle CDE &= 90^\circ - 24^\circ \text{ (complementary } \angle\text{s)} \\ &= 66^\circ\end{aligned}$$

Question 20

(a) $\angle ADC = 23^\circ$ (\angle in the same seg.)

(b) $\angle ODC = 50^\circ$ (isos. Δ)

$$\angle ADC = 23^\circ \text{ (}\angle\text{s in the same seg.)}$$

$$\begin{aligned}\angle ADO &= 50^\circ - 23^\circ \\ &= 27^\circ\end{aligned}$$

(c) $\angle OAD = 27^\circ$ (isos. Δ)

$$\angle QAD = 90^\circ \text{ (tan } \perp \text{ to radius)}$$

$$\begin{aligned}\angle QAD &= 90^\circ - 27^\circ \\ &= 63^\circ\end{aligned}$$

Question 23

(a)

$$\begin{aligned}CM &= 5 \div 2 \text{ (}\angle \text{ from centre bisects chord)} \\ &= 2.5 \text{ cm}\end{aligned}$$

(b)

$$\begin{aligned}OC^2 &= ON^2 + NC^2 \\ &= (3)^2 + (2.5)^2 \\ OC &= 3.9051, -3.9051 \text{ (reject)} \\ &= 3.91 \text{ cm (3 sig. fig.)}\end{aligned}$$

The radius is 3.91 cm.

Question 26

(b) (i) $\angle HEF = 46^\circ$ (\angle s in same segment)

(ii)

$$\begin{aligned}\angle AOC &= 90^\circ - 24^\circ \\ &= 66^\circ \text{ (sum of } \angle\text{s in } \Delta)\end{aligned}$$

$$\begin{aligned}\angle AFB &= 66^\circ \div 2 \\ &= 33^\circ \text{ (}\angle \text{ at centre} = 2 \times \angle \text{ at circumference)}\end{aligned}$$

(iii)

$$\begin{aligned}\angle HGF &= 180^\circ - 46^\circ \\ &= 134^\circ \text{ (opp. } \angle\text{s of cyclic quadrilateral)}\end{aligned}$$

(iv)

$$\begin{aligned}\angle FOD &= 180^\circ - 66^\circ \text{ (}\angle\text{s on a straight line)} \\ &= 114^\circ\end{aligned}$$

$$\text{Reflex } \angle FOD = 246^\circ \text{ (}\angle\text{s at a point)}$$

$$\begin{aligned}\angle FED &= 246^\circ \div 2 \quad (\angle \text{ at centre} = 2 \times \angle \text{ at circumference}) \\ &= 123^\circ\end{aligned}$$

Question 28

(a)

$$\begin{aligned}\angle DOE &= 25^\circ \times 2 \quad (\angle \text{ at centre} = 2 \times \angle \text{ at circumference}) \\ &= 50^\circ\end{aligned}$$

(b) $\angle BDO = 50^\circ$

$$\begin{aligned}\angle BOD &= 180^\circ - (50 \times 2) \quad (\text{isos. } \Delta) \\ &= 80^\circ\end{aligned}$$

(c)

$$\begin{aligned}\angle BAD &= \frac{80^\circ}{2} \quad (\angle \text{ at centre} = 2 \times \angle \text{ at circumference}) \\ &= 40^\circ\end{aligned}$$

(d)

$$\begin{aligned}\angle BDA &= 50^\circ - 25^\circ \quad (\text{isos. } \Delta) \\ &= 25^\circ\end{aligned}$$

$$\begin{aligned}\angle ABD &= 180^\circ - 40^\circ - 25^\circ \quad (\angle \text{ sum of } \Delta) \\ &= 115^\circ\end{aligned}$$

(e)

$$\begin{aligned}\angle BCD &= 180^\circ - 40^\circ \\ &= 140^\circ\end{aligned}$$

Question 30

(a) $\angle ABD = 52^\circ$ (\angle s in same segment)

(b) $\angle AOD = 104^\circ$ (\angle at centre = 2 \angle at circumference)

(c)

$$\begin{aligned}\angle ADO &= \frac{180^\circ - 104^\circ}{2} \quad (\text{base } \angle \text{ s of isos. } \Delta) \\ &= 38^\circ\end{aligned}$$

(d)

$$\begin{aligned}\angle ODC &= 90^\circ - 52^\circ \quad (\text{alt. } \angle \text{ s, tan } \perp \text{ rad}) \\ &= 38^\circ\end{aligned}$$

(e)

$$\begin{aligned}\angle CAE &= 180^\circ - (180^\circ - 52^\circ - 17^\circ) \quad (\text{opp. } \angle \text{ s of cyclic quad.}) \\ &= 69^\circ\end{aligned}$$

Question 31

(a)

$$\begin{aligned}\text{reflex } \angle BOC &= 360 - (70 \times 2) \quad (\angle\text{s at a point, } \angle \text{ at centre} = 2\angle \text{ at circle}) \\ &= 220^\circ\end{aligned}$$

(b) $\angle ABP = 70^\circ$ (ext. \angle of cyclic *quadrilateral*)

(c) $\angle MBN = 70^\circ$ (\angle in same segment)

(d)

$$\begin{aligned}\angle BMP &= 180 - (70 \times 2) \quad (\angle \text{ in opp. segment}) \\ &= 40^\circ\end{aligned}$$

(e)

$$\begin{aligned}\angle OCP &= \frac{180 - 62}{2} \quad (\angle \text{ sum of isos. } \triangle OPC) \\ &= 59^\circ\end{aligned}$$

$$\begin{aligned}\angle ACO &= 180 - 70 - 59 \quad (\text{adj. } \angle\text{s on straight line}) \\ &= 51^\circ\end{aligned}$$

Question 32

$$2x + 5y = 90 \quad - (1)$$

$$5x + 8y = 180 \quad - (2)$$

$$(1) \times 5, \quad 10x + 25y = 450 \quad - (3)$$

$$(2) \times 2, \quad 10x + 16y = 360 \quad - (4)$$

$$(3) - (4), \quad 9y = 90$$

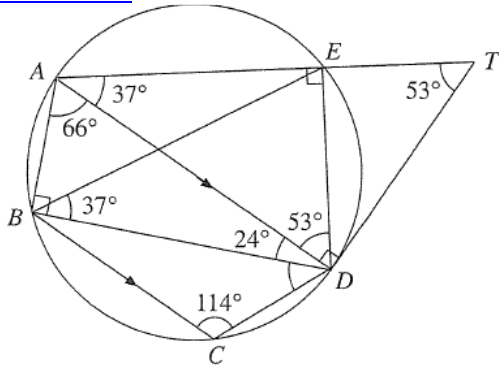
$$y = 10$$

Sub. $y = 10$ into (1),

$$2x + 5(10) = 90$$

$$x = 20$$

Question 33



(a) $\angle AED = 90^\circ$ (right \angle in semi - circle)

$$\begin{aligned}\angle ADE &= 180^\circ - 90^\circ - 37^\circ \\ &= 53^\circ \text{ (}\angle \text{ sum of } \Delta\text{)}\end{aligned}$$

(b) $\angle ADT = 90^\circ$ (tan \perp radius)

$$\begin{aligned}\angle ATD &= 180^\circ - 90^\circ - 37^\circ \\ &= 53^\circ\end{aligned}$$

(c)

$$\begin{aligned}\angle DBE &= \angle DAE \\ &= 37^\circ \text{ (}\angle\text{s in the same segment)}\end{aligned}$$

(d)

$$\begin{aligned}\angle BAD &= 180^\circ - \angle BCD \text{ (opp. } \angle\text{s of cyclic quadrilateral)} \\ &= 180^\circ - 114^\circ \\ &= 66^\circ\end{aligned}$$

(e) $\angle ABD = 90^\circ$ (right \angle in semi - circle)

$$\begin{aligned}\angle ADB &= 180^\circ - 90^\circ - 66^\circ \\ &= 24^\circ \text{ (}\angle \text{ sum of } \Delta\text{)}\end{aligned}$$

$$\begin{aligned}\angle BDC + 24^\circ + 114^\circ &= 180^\circ \text{ (int. } \angle\text{s, } AD \parallel BC\text{)} \\ \angle BDC &= 42^\circ\end{aligned}$$

Question 34

(a)

$$\begin{aligned}\angle BAD &= 180^\circ - 117^\circ \text{ (}\angle \text{ in opp. segments)} \\ &= 63^\circ\end{aligned}$$

(b)

$$\begin{aligned}\angle AOB &= 180^\circ - 2 \times 32^\circ \text{ (base } \angle\text{s of isos. } \Delta\text{)} \\ &= 116^\circ\end{aligned}$$

$$\begin{aligned}\angle ADB &= \frac{1}{2} \times 116^\circ \quad (\angle \text{ at centre} = \text{twice } \angle \text{ at circum.}) \\ &= 58^\circ\end{aligned}$$

(c)

$$\begin{aligned}\angle OBD &= 180^\circ - 63^\circ - 58^\circ - 31^\circ \quad (\angle \text{ sum of } \Delta) \\ &= 27^\circ\end{aligned}$$

Question 35

(a) $\angle CDE = 65^\circ$ (ext. \angle of cyclic. quad.)

(b)

$$\begin{aligned}\angle BCE &= 65 - 50 \\ &= 15^\circ \quad (\angle \text{ s in same segment})\end{aligned}$$

(c) $\angle CAE = \angle DAB$ (shared \angle)

$$\angle ECA = \angle BDA \quad (\angle \text{ s in same segment})$$

$$\angle AEC = \angle ABD \quad (\angle \text{ sum of } \Delta)$$

Triangle AEC is similar to triangle ABD by AAA

Question 36

(c) (i) angle $EBC = 61^\circ$ (base angles of isosceles triangle)

(ii) Angle $BEC = 29^\circ$ (angle at centre = 2 angle at circumference)

(iii) Angle $EDC = 119^\circ$ (angles in opp. segment)

(iv)

$$\begin{aligned}\text{Angle } CET &= 90^\circ - 29^\circ \quad (\text{tangent } \perp \text{ radius}) \\ &= 61^\circ\end{aligned}$$

(d)

$$\begin{aligned}\text{Angle } EOT &= \frac{180^\circ - 58^\circ}{2} \\ &= 61^\circ\end{aligned}$$

$$\tan 61^\circ = \frac{ET}{3}$$

$$ET = 3 \tan 61^\circ$$

$$\begin{aligned}\text{Area of } OETC &= 3 \tan 61^\circ \times 3 \\ &\approx 16.2 \text{ cm}^2\end{aligned}$$

Question 37

(a) (i) $\angle EAT = 36^\circ$ (alt. \angle s, $DE \parallel ST$)

$$\begin{aligned}\angle OAB &= 90^\circ - 36^\circ \text{ (tangent } \perp \text{ radius)} \\ &= 54^\circ\end{aligned}$$

(ii)

$$\begin{aligned}\angle ADE &= 180^\circ - 36^\circ - 83^\circ \text{ (}\angle \text{ sum of } \Delta\text{)} \\ &= 61^\circ\end{aligned}$$

$$\begin{aligned}\angle ABC &= 180^\circ - 61^\circ \text{ (}\angle \text{s in opp. segments)} \\ &= 119^\circ\end{aligned}$$

(iii)

$$\begin{aligned}\angle OAD &= 83^\circ - 54^\circ \\ &= 29^\circ\end{aligned}$$

$$\begin{aligned}\angle AOD &= 180^\circ - 2(29^\circ) \\ &= 122^\circ\end{aligned}$$

(b)

$$\begin{aligned}\angle ACD &= \frac{1}{2}(122^\circ) \text{ (}\angle \text{ at centre} = 2 \times \angle \text{ at circumference)} \\ &= 61^\circ\end{aligned}$$

Hence, ACD is isosceles. (shown)

Question 38

(a)

$$\begin{aligned}\angle DOE &= 180^\circ - 2(54^\circ) \text{ (Isosceles } \triangle DOE\text{)} \\ &= 72^\circ\end{aligned}$$

(b)

$$\begin{aligned}\angle DBE &= \frac{1}{2}\angle DOE \text{ (}\angle \text{ at centre} = 2\angle \text{ at circumference)} \\ &= 36^\circ\end{aligned}$$

(c) $\angle OEF = 90^\circ$ (tangent \perp to radius)

$$\begin{aligned}\angle DEF &= 90^\circ - 54^\circ \\ &= 36^\circ\end{aligned}$$

(d)

$$\begin{aligned}\angle BED &= 180^\circ - 116^\circ \text{ (}\angle \text{ in opp. segments)} \\ &= 64^\circ\end{aligned}$$

$$\begin{aligned}\angle OEB &= 64^\circ - 54^\circ \\ &= 10^\circ\end{aligned}$$

(e) $\angle BAE = \angle BAD + \angle DAE$

$$\angle DAE = 36^\circ \text{ (}\angle\text{s in same segment)}$$

$$\begin{aligned}\angle BAD &= 180^\circ - 116^\circ \\ &= 64^\circ\end{aligned}$$

$$\begin{aligned}\angle BAE &= 36^\circ + 64^\circ \\ &= 100^\circ\end{aligned}$$

Alternatively,

$$\begin{aligned}\angle BED &= 54^\circ + 10^\circ \\ &= 64^\circ\end{aligned}$$

$$\begin{aligned}\angle BDE &= 180^\circ - 64^\circ - 36^\circ \text{ (angle sum of triangle)} \\ &= 80^\circ\end{aligned}$$

$$\begin{aligned}\angle BAE &= 180^\circ - 80^\circ \text{ (angles in opposite segments)} \\ &= 100^\circ\end{aligned}$$

Question 39

(a) (i)

$$\begin{aligned}\angle AOB &= 180^\circ - 2(25^\circ) \\ &= 130^\circ \text{ (Isosceles triangle)}\end{aligned}$$

(ii)

$$\begin{aligned}\angle PAB &= 90^\circ - 25^\circ \\ &= 65^\circ \text{ (tangent } \perp \text{ to radius)}\end{aligned}$$

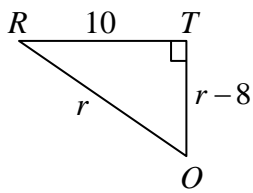
(iii)

$$\begin{aligned}\angle APB &= 360^\circ - 180^\circ - 130^\circ \\ &= 50^\circ \text{ (sum of angles in quadrilateral)}\end{aligned}$$

Or

$$\begin{aligned}\angle APB &= 180^\circ - 2(65^\circ) \\ &= 50^\circ \text{ (tangents from ext. point)}\end{aligned}$$

Question 41



$$r^2 = (r-8)^2 + 10^2$$

$$r^2 - r^2 + 16r = 64 + 100$$

$$r = \frac{164}{16}$$

$$r = 10.25 \text{ cm or } 10\frac{1}{4} \text{ cm}$$

Question 42

(a) (i) $\angle BDA = 24^\circ$ (\angle s in same segment)

$$\begin{aligned}\angle BAD &= 180^\circ - 90^\circ - 24^\circ \\ &= 66^\circ \text{ (\angle s in semicircle)}\end{aligned}$$

(ii) $\angle BED = 66^\circ$ (\angle s in same segment)

(iii) $\angle BOQ = 48^\circ$ (\angle at centre = 2 \angle at circumference)

$$\begin{aligned}\angle BQA &= 180^\circ - 90^\circ - 48^\circ \\ &= 42^\circ \text{ (tangent } \perp \text{ radius)}\end{aligned}$$

(iv)

$$\begin{aligned}\text{Reflex } \angle BOD &= 180^\circ + 48^\circ \\ &= 228^\circ\end{aligned}$$

(v)

$$\begin{aligned}\angle ABQ &= 90^\circ - 66^\circ \text{ (tangent } \perp \text{ radius)} \\ &= 24^\circ\end{aligned}$$

(b)

$$\begin{aligned}\angle ADE &= 180^\circ - 90^\circ - x \\ &= 90^\circ - x \text{ (\angle in semicircle)}\end{aligned}$$

$$\angle ABE = 90^\circ - x$$

$$\begin{aligned}\angle QBE &= 90^\circ - x + 24^\circ \\ &= (114 - x)^\circ \text{ (Shown)}\end{aligned}$$

(c)

$$\begin{aligned}\angle EBO &= 66^\circ - 43^\circ \\ &= 23^\circ\end{aligned}$$

BO and AE are not parallel as $\angle EBQ \neq \angle AEB$.

Question 43

(a) $\angle OCE = 90^\circ$ (radius \perp tangent)

(b)

$$\begin{aligned}\angle DOC &= 180^\circ - 90^\circ - 28^\circ \\ &= 62^\circ\end{aligned}$$

$$\begin{aligned}\angle ODC &= \frac{180^\circ - 62^\circ}{2} \\ &= 59^\circ \text{ (isos. } \Delta\text{)}\end{aligned}$$

(c)

$$\begin{aligned}\angle DAC &= 62^\circ \div 2 \\ &= 31^\circ \text{ (}\angle \text{ at centre} = 2 \angle \text{ at circumference)}\end{aligned}$$

(d) $\angle DBC = 31^\circ$ (\angle in same segment)

(e) $\angle ABD = \angle ACD$ (\angle in same segment)

$$\begin{aligned}\angle DCB &= \angle DCO + \angle OCB \\ &= 90^\circ \text{ (}\angle \text{ in semicircle)}\end{aligned}$$

$$\begin{aligned}\angle ACB &= 90^\circ - 35^\circ \\ &= 55^\circ\end{aligned}$$

Question 44

(a)

$$\begin{aligned}\angle RTS &= \frac{50}{2} \text{ (}\angle \text{ at centre} = 2 \times \angle \text{ at circumference)} \\ &= 25^\circ\end{aligned}$$

(b) $\angle PST = 34^\circ$ (\angle s in same segment are equal)

$$\begin{aligned}\angle OSP &= \frac{50}{2} \text{ (ext. } \angle \text{ of } \Delta \text{ or isos. } \Delta\text{)} \\ &= 25^\circ\end{aligned}$$

$$\begin{aligned}\angle OST &= 34 + 25 \\ &= 59^\circ\end{aligned}$$

(c) $\angle QPR = 34^\circ$ (alt. \angle s)

$$\begin{aligned}\angle QRP &= 180 - 90 - 34 \quad (\angle \text{ in semicircle is } 90^\circ) \\ &= 56^\circ\end{aligned}$$

(d) $\triangle OPU$ is right angled.

$$\begin{aligned}\angle OPU &= \frac{130^\circ}{2} \\ &= 65^\circ\end{aligned}$$

$$\begin{aligned}\tan 65^\circ &= \frac{PU}{5} \\ PU &= 5 \tan 65^\circ \\ &= 10.7 \text{ cm (3 sig. fig.)}\end{aligned}$$

Question 45

(a) (i) $\angle ACE = 26^\circ$ (\angle s in the same segment)

$$\begin{aligned}\text{(ii)} \quad \angle CBE &= 90^\circ - 26^\circ \\ &= 64^\circ \quad (\angle \text{ in a semicircle})\end{aligned}$$

$$\begin{aligned}\text{(iii)} \quad \angle CDE &= 180^\circ - 64^\circ \\ &= 116^\circ \quad (\angle \text{ s in opp. segments})\end{aligned}$$

(iv) $\angle CAE = 64^\circ$ (\angle s in the same segment)

$$\begin{aligned}\angle AEQ &= 90^\circ - 64^\circ \quad (\text{tan } \perp \text{ rad}) \\ &= 26^\circ\end{aligned}$$

Or

$$\angle AEQ = 26^\circ \quad (\angle \text{ in alt. segment})$$

(b)

$$\begin{aligned}\angle OEB &= 64^\circ - 38^\circ \\ &= 26^\circ\end{aligned}$$

$$\begin{aligned}\angle OEB &= \angle ABE \\ &= 26^\circ\end{aligned}$$

Hence, lines EO and AB are parallel since $\angle OEB$ and $\angle ABE$ are alternate angles.

Question 46

(a) (i) $\angle CDE = 120^\circ$ (\angle at center = $2 \times \angle$ at circumference)

$$\begin{aligned}\angle DCE &= \frac{180^\circ - 120^\circ}{2} \text{ (isosceles } \Delta) \\ &= 30^\circ\end{aligned}$$

$$\begin{aligned}\text{(ii)} \quad \angle CBE &= 180^\circ - 120^\circ \text{ (opp. } \angle\text{s of cyclic quad.)} \\ &= 60^\circ\end{aligned}$$

$$\begin{aligned}\text{(iii)} \quad \angle BCE &= 180^\circ - 100^\circ \text{ (opp. } \angle\text{s of cyclic quad.)} \\ &= 80^\circ\end{aligned}$$

$$\begin{aligned}\angle CEB &= 180^\circ - 80^\circ - 60^\circ \text{ (} \angle \text{ sum of } \Delta) \\ &= 40^\circ\end{aligned}$$

$$\begin{aligned}\text{(b)} \quad \angle BCT &= 180^\circ - 20^\circ - (180^\circ - 60^\circ) \text{ (} \angle \text{ sum of } \Delta) \\ &= 40^\circ\end{aligned}$$

$$\begin{aligned}\angle BCO &= 80^\circ - \frac{1}{2}(180^\circ - 120^\circ) \text{ (isosceles } \Delta) \\ &= 50^\circ\end{aligned}$$

$$\begin{aligned}\angle OCT &= 40^\circ + 50^\circ \\ &= 90^\circ\end{aligned}$$

Hence by tangent perpendicular radius, CT is a tangent to the circle at C .

Question 47

$$\begin{aligned}\text{(a) (i)} \quad \angle BOE &= 2 \times 70^\circ \text{ (} \angle \text{ at centre} = 2 \times \angle \text{ at circumference)} \\ &= 140^\circ\end{aligned}$$

$$\begin{aligned}\text{(ii)} \quad \angle BDE &= 180 - 140 \text{ (opp. } \angle\text{s of cyclic quad)} \\ &= 40^\circ\end{aligned}$$

$$\begin{aligned}\text{(iii)} \quad \angle ABD &= 180 - 70 - 40 \text{ (} \angle \text{ sum of } \Delta) \\ &= 70^\circ\end{aligned}$$

$$\begin{aligned}\text{(iv)} \quad \angle CBD &= 180 - 70 \text{ (} \angle\text{s on a straight line)} \\ &= 110^\circ\end{aligned}$$

$$\angle CED = 110^\circ \text{ (} \angle\text{s in same segment)}$$

$$\begin{aligned}
 \text{(v)} \quad \angle DFC &= 40 + 110 \quad (\text{ext. } \angle \text{ of a } \Delta) \\
 &= 150^\circ
 \end{aligned}$$

Question 48

(a) By Pythagoras' Theorem,

$$PO = \sqrt{12^2 + 5^2}$$

$$PO = 13$$

$$m = PO - 5$$

$$m = 13 - 5$$

$$= 8$$

(b)

$$\tan \angle BPO = \frac{5}{12}$$

$$\angle BPO = \tan^{-1}\left(\frac{5}{12}\right)$$

$$\angle BPA = 2 \times \angle BPO$$

$$= 2 \times \tan^{-1}\left(\frac{5}{12}\right)$$

$$= 45.2397^\circ$$

$$= 45.2^\circ \quad (\text{to 1 decimal place})$$

Question 49

(a)

$$\angle AOD = 2\angle ACD \quad (\angle \text{ at centre} = 2 \angle \text{ at circumference})$$

$$\angle AOD = 2(46^\circ)$$

$$\angle AOD = 92^\circ$$

$$\angle ADO = \frac{(180^\circ - 92^\circ)}{2} \quad (\text{base } \angle \text{ s of isos. } \Delta)$$

$$\angle ADO = 44^\circ$$

(b)

$$\angle ABD = \angle ACD$$

$$= 46^\circ \quad (\angle \text{ in same segment})$$

$$\angle AFB = 180^\circ - 46^\circ - 25^\circ$$

$$\angle AFB = 109^\circ \quad (\angle \text{ sum of } \Delta)$$

$$\angle OFD = 180^\circ - 109^\circ$$

$$\angle OFD = 71^\circ \quad (\angle \text{ on a st. line})$$

(c) $\angle CAE = 90^\circ$ (radius \perp tangent)

$$\angle AEC = 180^\circ - 46^\circ - 90^\circ \quad (\angle \text{ sum of } \Delta)$$

$$\angle AEC = 44^\circ$$

Question 50

(a) (i) $\angle OAG = 90^\circ$ (tan \perp rad)

$$\begin{aligned}\angle GOA &= 180^\circ - 30^\circ - 90^\circ \quad (\angle \text{ sum of } \Delta) \\ &= 60^\circ\end{aligned}$$

(ii)

$$\begin{aligned}\angle FDA &= \frac{1}{2} \times 60^\circ \quad (\angle \text{ at centre} = 2 \angle \text{ at circumference}) \\ &= 30^\circ\end{aligned}$$

(iii) $\angle OAD = 30^\circ$ (base \angle of isos. Δ)

$$\begin{aligned}\angle BAD &= 180^\circ - 110^\circ \quad (\angle \text{ s in opp. segments}) \\ &= 70^\circ\end{aligned}$$

$$\begin{aligned}\angle BAO &= 70^\circ - 30^\circ \\ &= 40^\circ\end{aligned}$$

(iv) $\angle FCD = 90^\circ$ (\angle in semicircle)

$$\begin{aligned}\angle BCF &= 110^\circ - 90^\circ \\ &= 20^\circ\end{aligned}$$

$$\angle BDF = 20^\circ \quad (\angle \text{ s in the same segment})$$

(b)

$$\begin{aligned}\angle OFA &= \angle OAF \\ &= \angle AOF \\ &= 60^\circ\end{aligned}$$

Therefore, $\triangle OAF$ is an equilateral triangle.

$$\begin{aligned}OF &= OA \\ &= AF\end{aligned}$$

In $\triangle GAO$ and $\triangle DAF$,

$$\angle OAG = 90^\circ \text{ (tan } \perp \text{ rad)}$$

$$\angle FAD = 90^\circ \text{ (}\angle \text{ in semicircle)}$$

$$\begin{aligned}\angle OAG &= \angle FAD \\ &= 90^\circ \text{ (A)}\end{aligned}$$

$$\begin{aligned}\angle AGO &= \angle ADF \\ &= 30^\circ \text{ (A)}\end{aligned}$$

$$OA = FA \text{ (equilateral triangle) (S)}$$

$$\therefore \triangle GAO \equiv \triangle DAF \text{ (AAS) (Shown)}$$